per cent. more than the nitrate. We have shown that sodium sulphate should be hydrolyzed to a greater extent than sodium nitrate; consequently, it should have not only a greater solvent action than the nitrate, but also a greater decomposing action on the organic matter, resulting in the production of a darker brown color. In view of the very general and abundant distribution of alkali sulphates in western soils, and granting the presence of sufficient organic matter for the production of a brown color according to the hypothesis of Stewart and Peterson, we should expect to find a uniform distribution of a dark brown surface color throughout the soils of the arid west. This is not the case.

As a matter of fact, neither the nitrates nor the sulphates are hydrolyzed at all. Stewart and Peterson admit that the sulphates are not hydrolyzed, but claim that the nitrates are. It is indeed difficult to reconcile their claim with the facts in the case. Since the alkali nitrates are not hydrolyzed, and since alkali hydroxides could be produced from them in no other way than by their hydrolysis, it is very evident that no alkali hydroxides are formed, and consequently the explanation of the brown color of the "niter spots" as given by Stewart and Peterson is nothing short of preposterous.

The question raised by the above writers concerning the relation of the pigment of *Azotobacter chroococcum* and the nitrate to the brown color of the "niter spots" applies only to the colorless strains of the organism. These have been shown by us to be capable of producing abundant brown to black pigment when supplied with very small quantities of nitrate (0.01 to 0.03 per cent. sodium nitrate) and some source of energy, both of which are present in our niter soils. Aside from these colorless strains, their contention is wholly irrelevant for we have already pointed out[1] that four of the seven strains of *A. chroococcum*, isolated from niter soils have produced, at one time or another, pigments varying in color from a delicate cream, through the different shades of brown, to an intensive brownish black in the *total absence of nitrates*, thereby attesting their ability to produce the pigment independent of the nitrate.

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[1] Colorado Experiment Station Bulletin, 179, p. 33, June, 1911.

CALCULATION OF THE GAMMA FUNCTION

TO THE EDITOR OF SCIENCE: In SCIENCE, April 2, 1915, Dr. Raymond Pearl has described a method of obtaining an approximate value of $\log \Gamma(n)$ by interpolating in a table of log-factorial n.

In a table he compares the values obtained by interpolation, by Forsyth's method of approximation, and those found by using the relation

$$\Gamma(n+1) = n(n-1)(n-2)\cdots(n-r)\Gamma(n-r)$$

in conjunction with Legendre's tables, calling the value obtained by the last method the "exact" value.

Dr. Pearl explains that this "exact" value derived its name from the exactness of the mathematical argument upon which it is based, and not from any special accuracy in the numerical values given under that heading and he disclaims any greater degree of accuracy than can be expected from the summation of a large number of seven-place logarithms.

As the subject to be investigated is not how closely the values obtained by different methods agree with one another but how closely the approximate to the true value of the function, I have calculated a "more exact" value for the cases given by Dr. Pearl, using Legendre's table and ten-place logarithms and I believe the values so obtained are correct within a half-unit of the seventh decimal place.

Another set of values has also been calculated by means of Pearson's approximation formula[1]

[1] "On a Formula for Determining $\Gamma(x+1)$," *Biometrika*, Vol. VI., p. 118.

$$\log \frac{\Gamma(x+1)}{x^x e^{-x}} = .3990899 + \tfrac{1}{2}\log x + .080929 \sin \frac{25°.623}{x}$$

or

$$\log \Gamma(x+1) = .6162372 + (x+\tfrac{1}{2})(\log x - \log e) + .080929 \sin \frac{25°.623}{x},$$

using seven-place logarithms except for the second or product term where ten-place logarithms were used to avoid introducing inaccuracies when x is large. This formula is also given in the introduction to "Tables for Statisticians and Biometricians" (Chicago University Press), on page lv, where unfortunately by a printer's error the value 0.3990899 is wrongly given as .03990899.

The various series of values are summarized in the following table.

VALUES OF LOG $\Gamma(n)$ BY DIFFERENT METHODS

n	Pearl's "Exact" Value	Pearl Forsyth	Pearl Using Δ^2	Pearl Using Δ^3	Pearson	"More Exact" Value
5.123	1.4613860	1.4613679	1.4619138	1.4615009	1.4613859	1.4613860
15.123	11.0834931	11.0834916	11.0835559	11.0834985	11.0834931	11.0834930
25.123	23.9637108	23.9637096	23.9637336	23.9637119	23.9637107	23.9637107
35.123	38.6594135	38.6594126	38.6594251	38.6594138	38.6594133	38.6594133
75.123	107.7498704	107.7498692	107.7498727	107.7498702[2]	107.7498702	107.7498702

The table shows that Dr. Pearl's "exact" value differs from the "more exact" value by two units in the seventh place for the larger values of n and in the case of $n = 75.123$ is inferior to the value found from interpolation when third differences are used.

A comparison of the values in the table leads to the following conclusions.

(i) For small values of n, up to about 5, it is preferable to use the exact method if Legendre's tables are available; in the absence of Legendre's tables the Pearson approximation formula should be used.

(ii) For larger values of n, as shown by the middle portion of the table, Pearson's formula is superior to the interpolation method and gives results which coincide with those found by the

(iii) For still larger values of n, 75 and upwards, the Pearson approximation formula and the interpolation method using third differences both give the true value to the seventh decimal place, but while the usefulness of the interpolation method is limited by the range of the existing tables of log-factorial n, that of the approximation method is not affected, provided a sufficient number of places be used in the logarithms of x and e when computing the second or product term.

P. F. Everitt

[2] Given as 107.7492870 in Dr. Pearl's article, being a misprint for the value given above, which I verified by recalculation.

THE POSITION OF REFERENCES IN JOURNAL ARTICLES

From one half to one per cent. of the space in the majority of scientific journals giving many references is wasted by the faulty position and arrangement of the references.

The amount of time wasted by the reader will depend on whether he is obliged to look up the references, or simply glances at them occasionally to see a date, or the name of an *author or journal.*

The word reference is defined here to mean the author's name, journal title (usually abbreviated), with the numbers for series, volume, pages and date. If any information from the original is also given, and printed at the bottom of a page outside the text, the whole is regarded as a foot-note, and is not considered here. "*Loc. cit.*," is regarded as a reference.

Most journals are printed with a solid page, at the foot of which are the references for that page, with the reference numbers indicated in the text, a separate line being given to each reference, except where extra space is required either because of grouping several references under one number, or because of unusual length of names.

Nearly one per cent. of the space used in printing articles and references in this way can be saved by giving each reference a number (the numbers to run consecutively), then printing all the references at the end of the article, leaving an extra-wide spacing between the period at the end of one reference and the next number, in order to catch the eye. There